SIGRAV Lecture Notes: Cosmological Tests of Gravity

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1 Cosmological Perturbation Theory

1.1 Metric

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} \tag{1}$$

Perturbed metric:

$$ds^2 = -a^2(1-2A)\,d\eta^2 + a^2\partial_i B dx^i d\eta + a^2\left[(1+2C)\gamma_{ij} + D_{ij}E\right]dx^i dx^j \tag{2}$$

$$ds^2 = -a^2(1+2\Psi)\,d\eta^2 + a^2(1-2\Phi)\gamma_{ij}dx^i dx^j \tag{3}$$

where Φ, Ψ are functions of η, x, y, z. Hubble factor $\mathcal{H} = \dot{a}/a$, dot represents conformal time derivative.

$$\bar{G}_{00} = 3\mathcal{H}^2 \tag{4}$$

$$\bar{G}_{ij} = -\delta_{ij}\left(2\dot{\mathcal{H}} + \mathcal{H}^2\right) \tag{5}$$

$$\delta G_{00} = 2\nabla^2\Phi - 6\mathcal{H}\dot{\Phi} \tag{6}$$

$$\delta G_{0i} = 2\nabla_i\left(\dot{\Phi} + \mathcal{H}\Psi\right) \tag{7}$$

$$\delta G_{ij} = \delta_{ij}\left[2(\Phi+\Psi)(2\dot{\mathcal{H}} + \mathcal{H}^2) + 2\mathcal{H}(2\dot{\Phi} + \dot{\Psi}) + 2\ddot{\Phi} - \delta^{km}\partial_k\partial_m(\Phi - \Psi)\right] + \partial_i\partial_j(\Phi - \Psi) \tag{8}$$

$$\Rightarrow \delta G_{ij} = \partial_i\partial_j(\Phi - \Psi) \qquad \text{for i} \neq \text{j} \tag{9}$$

1.2 Energy-momentum Tensor

Stress-energy tensor of fluid, four velocity u^μ satisfying $u^\mu u_\mu = -1$:

$$T_{\mu\nu} = (\rho + P)u_\mu u_\nu + P g_{\mu\nu} \tag{10}$$

$$u^\mu u_\mu = g_{\mu\nu}u^\nu u^\mu = -a^2(u^0)^2 = -1 \tag{11}$$

$$\Rightarrow \bar{u}^0 = \frac{1}{a} \quad \text{and} \quad \bar{u}_0 = g_{\mu 0}u^\mu = -a \tag{12}$$

where the last line follows because for a comoving observer, $u_i = 0$.

$$\Rightarrow \bar{T}_{00} = (\rho + P)u_0^2 + P g_{00} = a^2(\rho + P) - a^2 P = a^2\rho \tag{13}$$

$$\bar{T}_{ij} = P g_{ij} = a^2 P\delta_{ij} \tag{14}$$

Now perturb:

$$\delta g_{\mu\nu}\bar{u}^\mu\bar{u}^\mu + 2\bar{g}_{\mu\nu}\bar{u}^{(\mu}\delta u^{\nu)} = 0 \tag{15}$$

$$\Rightarrow \delta g_{00}\bar{u}^0\bar{u}^0 + 2\bar{g}_{00}\bar{u}^{(0}\delta u^{0)} = -2a^2\Psi \cdot \frac{1}{a^2} - 2a^2 \cdot \frac{1}{a} \cdot \delta u^0 = 0 \tag{16}$$

$$\Rightarrow \delta u^0 = -\frac{\Psi}{a} \tag{17}$$

Let $\delta u^i = \dfrac{v^i}{a}$, v^i is the three velocity.

$$\delta u^\mu = \frac{1}{a}(-\Psi, v^i) \qquad \text{and} \qquad \delta u_\mu = a(-\Psi, v_i) \tag{18}$$

Now for the energy momentum tensor:

$$\delta T_{\mu\nu} = (\delta\rho + \delta P)\bar{u}_\mu \bar{u}_\nu + 2(\rho + P)\delta u_{(\mu} u_{\nu)} + \delta P g_{\mu\nu} + P\delta g_{\mu\nu} + a^2 P\,\Pi_{\mu\nu} \tag{19}$$

$$\delta T_{00} = a^2\rho(\delta + 2\Psi) \tag{20}$$

$$\delta T_{0i} = -a^2(\rho + P)v_i \qquad \text{where} \qquad v_i = \partial_i v + \hat{v}_i \tag{21}$$

$$\begin{aligned}\delta T_{ij} &= a^2\left[\delta P - 2P\Phi\right]\delta_{ij} + a^2 P\Pi_{ij} \\ &= a^2 P\left[(\delta P/P - 2\Phi)\delta_{ij} + \Pi_{ij}\right]\end{aligned} \tag{22}$$

where $\Pi_{ij} = \Delta_{ij}\Pi + \nabla_{(i}\Pi_{j)} + \hat{\Pi}_{ij}$, and the operator $\Delta_{ij} = \nabla_i\nabla_j - \delta_{ij}\nabla^2/3$ vanishes for $i = j$, i.e. off-diagonal only.

1.3 Einstein Equations

Equating components, we reach the cosmological Einstein equations perturbed up to linear order (where $\rho \equiv \Sigma_i \rho_i$):

$$3\mathcal{H}^2 = 8\pi G_N a^2 \rho \tag{23}$$

$$-(2\dot{\mathcal{H}} + \mathcal{H}^2) = 8\pi G_N a^2 P \tag{24}$$

$$2\nabla^2\Phi - 6\mathcal{H}\dot{\Phi} = 8\pi G_N \rho a^2(\delta + 2\Psi) \tag{25}$$

$$2\nabla_i\left(\dot{\Phi} + \mathcal{H}\Psi\right) = -8\pi G_N a^2(\rho + P)v_i \tag{26}$$

$$\Rightarrow 2\nabla^2\left(\dot{\Phi} + \mathcal{H}\Psi\right) = 8\pi G_N a^2(\rho + P)\theta \tag{27}$$

$$i \neq j \qquad \partial_i\partial_j(\Phi - \Psi) = 8\pi G_N a^2 P\Delta_{ij}\Pi \simeq 0 \qquad \Rightarrow \Phi = \Psi \tag{28}$$

where we have taken a covariant derivative of the $0i$ equation, and defined $\theta = \nabla^i v_i = \nabla^2 v$. Going glibly to Fourier space, this gives $v = -\theta/k^2$.

Pulling the derivatives off eq.(26) and recalling $v_i = \partial_i v$ (dropping vector part), we take the combination $00 + 3\mathcal{H}\nabla^{-1}(0i)$:

$$2\nabla^2\Phi + 6\mathcal{H}^2\Psi = 8\pi G_N \rho a^2[\delta - 3\mathcal{H}(1 + w)v + 2\Psi] \tag{29}$$

$$\Rightarrow 2\nabla^2\Phi = 8\pi G_N \rho a^2 \Delta \tag{30}$$

where $\Delta = \delta - 3\mathcal{H}(1 + w)v = \delta + 3\mathcal{H}(1 + w)\theta/k^2$ is a convenient *total matter* variable. On sub-horizon scales, $\mathcal{H}/k << 1$ and $\Delta \simeq \delta$.

Equations 28 and 30 are very important – they are two constraint equations in GR. It is almost a defining feature of MG that these equations get modified in other gravity theories. They tell us how the spacetime potentials are linked to each other and the perturbations of the matter content. However, to evolve the system forwards in time, we need the (perturbed) conservation of energy-momentum.

1.4 Fluid Equations

$$\delta(\nabla_\mu T^\mu_\nu) = 0 \tag{31}$$

Exercise: expand this in perturbed variables, including perturbed Christoffels (get these from my Mathematica notebook). Using the background conservation equation, $\dot{\rho} = -3\mathcal{H}(\rho + P)$, and defining the sound speed $c_s^2 = \delta P/\delta\rho$, show that the two components of eq.(31) yield:

$$\dot{\delta} + 3\mathcal{H}(c_s^2 - w)\delta + (1 + w)(\theta - 3\dot{\Phi}) = 0 \tag{32}$$

$$\dot{\theta} + \left[\mathcal{H}(1 - 3w) + \frac{\dot{w}}{1 + w}\right]\theta + \nabla^2\left(\frac{c_s^2}{1 + w}\delta + \Psi\right) = 0 \tag{33}$$

For non-relativistic matter, $c_s^2 = w = 0$:

$$\dot{\delta} + \theta - 3\dot{\Phi} = 0 \tag{34}$$

$$\dot{\theta} + \mathcal{H}\theta + \nabla^2\Psi = 0 \tag{35}$$

The $\dot{\Phi}$ term above is absent in Newtonian gravity. This equation says: the velocity (divergence) increases in the opposite direction to potential gradients, i.e. stuff falls into potential wells. To eliminate, take derivative of Poisson equation in and use $\Delta \sim \delta$ on scales of interest:

$$-2k^2\dot{\Phi} = 8\pi G_N \rho a^2[\dot{\delta} + 2\mathcal{H}\delta + \dot{\rho}/\rho] = 8\pi G_N \rho a^2[\dot{\delta} - \mathcal{H}\delta] \tag{36}$$

Use in eq.(35):

$$\dot{\delta} = -\theta - \frac{3}{2k^2}\left(8\pi G_N \rho a^2[\dot{\delta} - \mathcal{H}\delta]\right) = -\theta - \frac{9\mathcal{H}^2}{2k^2}[\dot{\delta} - \mathcal{H}\delta] \simeq -\theta \tag{37}$$

where the last line follows from $\mathcal{H}/k << 1$. Think about what this says: if there is a net inflow of matter into a volume, such that θ is negative, then δ will grow. Differentiate 37 and sub in θ eq:

$$\ddot{\delta} = -\dot{\theta} = \mathcal{H}\theta + \nabla^2\Psi \tag{38}$$
$$= -\mathcal{H}\dot{\delta} + \nabla^2\Psi \tag{39}$$
$$\Rightarrow \ddot{\delta} + \mathcal{H}\dot{\delta} - \frac{3\mathcal{H}^2\Omega_M}{2}\delta = 0 \tag{40}$$

where the last line has crucially used that $\Phi = \Psi$ in GR at late times.

Exercise: Show that, *during a matter-dominated era* ($\Omega_M \simeq 1$), the solutions of eq.(40) give $\delta \propto a$ and $\delta \propto a^{-3/2}$ (decaying mode).

The final important quantity we need to introduce is the *growth rate*. It is defined as (mixing up z and a):

$$f(z) = \frac{d\ln\Delta}{d\ln a} \simeq \frac{d\ln\delta}{d\ln a} \tag{41}$$

Then during a matter era, we have $f \simeq 1$. On subhorizon scales it is independent of k. See plot. Note surveys generally measure $f\sigma_8$.

2 MG Models

$$S_{GR} = \int d^4x\,\sqrt{-g}\left\{\frac{M_P^2}{2}(R-2\Lambda) - \mathcal{L}_M(\psi, g_{\mu\nu})\right\} \tag{42}$$

2.1 (Simple) Scalar-Tensor & $f(R)$ Actions

$$S_{ST} = \int d^4x\,\sqrt{-g}\left\{\frac{M_P^2}{2}\phi\,R - \frac{\omega(\phi)}{\phi}\nabla_\mu\phi\nabla^\mu\phi - 2U(\phi) - \mathcal{L}_M(\psi, g_{\mu\nu})\right\} \tag{43}$$

Note that we could write the coupling to the Einstein-Hilbert term as $F(\phi)$, but we are always able to redefine the scalar field so that we only need two out of $F(\phi)$, $\omega(\phi)$, $U(\phi)$. Classic JBD recovered in the limit $\omega =$ const., with the GR limit $\omega_{JBD} \to \infty$. The current bound from Solar System tests (Bertotti et al. 2003) is $\omega_{JBD} > 40,000$ (Gaia improvements?)

Field equations:

$$\phi G_{\mu\nu} + g_{\mu\nu}\left[\Box\phi + \frac{1}{2}\frac{\omega(\phi)}{\phi}(\nabla\phi)^2 + U(\phi)\right] - \nabla_\mu\phi\nabla_\nu\phi - \frac{\omega(\phi)}{\phi}\nabla_\mu\nabla_\nu\phi = 8\pi G_N T^m_{\mu\nu} \tag{44}$$

$$[2\omega(\phi)+3]\Box\phi + \partial_\phi\omega(\phi)\nabla_\mu\phi\nabla^\mu\phi + 4U(\phi) - 2\phi\partial_\phi U = 8\pi G T^m \tag{45}$$

$$\nabla_\mu[(T^m)^\mu_\nu] = 0 \tag{46}$$

Note that the gravitational part of the action can be made equivalent to GR under a conformal transformation:

$$g_{\mu\nu} = A^2(\phi)\tilde{g}_{\mu\nu} \qquad g^{\mu\nu} = \frac{1}{A^2(\phi)}\tilde{g}^{\mu\nu} \tag{47}$$

$$\sqrt{-g} = A^4(\phi)\sqrt{-\tilde{g}} \qquad R = \frac{1}{A^2(\phi)}\left[\tilde{R} - 6\tilde{\nabla}_\mu \ln A\tilde{\nabla}^\mu \ln A\right] \tag{48}$$

$$T_{\mu\nu} = \frac{1}{A^2(\phi)}\tilde{T}_{\mu\nu} \qquad T^{\mu\nu} = \frac{1}{A^6(\phi)}\tilde{T}^{\mu\nu} \tag{49}$$

where A is a factor we choose. We won't go through the algebra here (**exercise**), but to remove the coupling ϕR in the ST action, choose $A = \phi^{-1/2}$. After redefining the scalar field and potential according to

$$\frac{d\varphi}{d\ln a} = (12 + 8A^2\omega)^{1/2} \qquad V = 2A^4U(\phi) \tag{50}$$

we transform S_{ST} to

$$S_{ST} = \int d^4x\sqrt{-g}\left\{\frac{M_P^2}{2}\tilde{R} - \frac{1}{2}\tilde{\nabla}_\mu\varphi\tilde{\nabla}^\mu\varphi - V(\phi) - \mathcal{L}_M(\psi, A^2(\phi)\tilde{g}_{\mu\nu})\right\} \tag{51}$$

$$\Rightarrow G_{\mu\nu} = 8\pi G_N\left(\tilde{T}^m_{\mu\nu} + \tilde{T}^\varphi_{\mu\nu}\right) \tag{52}$$

$$\text{BUT} \quad \nabla_\mu(\tilde{T}^{\mu\nu}_m) \neq 0 \tag{53}$$

We won't pursue this generalised ST theory further (arbitrariness in potentials etc). Instead let's look at one of the most popular variants of ST, $f(R)$ gravity. Back to the Jordan frame action eq.(43), and set

$$\phi = \frac{df}{dR} = f_R \qquad \omega(\phi) = 0 \qquad U(\phi) = \frac{M_P^2}{4}\left[Rf_R - f(R)\right] \tag{54}$$

$$S_f = \int d^4x\,\sqrt{-g}\left\{\frac{M_P^2}{2}\left(f_R\,R - [Rf_R - f(R)]\right) - \mathcal{L}_M(\psi, g_{\mu\nu})\right\} \tag{55}$$

$$= \int d^4x\,\sqrt{-g}\left\{\frac{M_P^2}{2}f(R) - \mathcal{L}_M(\psi, g_{\mu\nu})\right\} \tag{56}$$

So $f(R)$ gravity is really just an ST theory, where the new degree of freedom is the 'scalaron' field, $\phi \sim f_R$.

2.2 $f(R)$ Background Cosmology

$$f_R R_{\mu\nu} - \frac{1}{2} f(R) g_{\mu\nu} - \nabla_\mu \nabla_\nu f_R + g_{\mu\nu} \Box(f_R) = 8\pi G_N T^m_{\mu\nu} \tag{57}$$
$$3\Box f_R + f_R R - 2f(R) = 8\pi G_N T^m \tag{58}$$

Now put this on a (unperturbed) FRW metric (where the second equation comes from taking the 00+ii components):

$$3f_R \mathcal{H}^2 = 8\pi G_N a^2 \rho_m + \frac{a^2}{2}[f_R R - f(R)] - 3\mathcal{H}\dot{f}_R \tag{59}$$
$$\Box f_R = \frac{8\pi G_N}{3}(3P_m - \rho_m) + \frac{2}{3} f(R) - \frac{1}{3} f_R R \equiv \frac{\partial V_{\text{eff}}}{\partial f_R} \tag{60}$$

$$-2(\dot{\mathcal{H}} - \mathcal{H}^2) = 8\pi G_N a^2 \rho_m (1 + w_m) + \ddot{f}_R - 2\mathcal{H}\dot{f}_R \tag{61}$$

Matter and the form of $f(R)$ act to form an effective potential for the scalaron. The effective mass of the scalaron is then:

$$m_\phi^2 = \frac{\partial^2 V_{\text{eff}}}{\partial f_R^2} = \frac{1}{3}\left(2\frac{df}{df_R} - R - f_R \frac{dR}{df_R}\right) \tag{62}$$
$$= \frac{1}{3}\left(2f_R \frac{dR}{df_R} - R - f_R \frac{dR}{df_R}\right) \tag{63}$$
$$= \frac{1}{3}\left(\frac{f_R}{f_{RR}} - R\right) \tag{64}$$

In order not to mess up the early universe, we generally want $f(R) \sim R$ at high R, and correspondingly $f_R \sim 1$, $f_{RR} << 1$.

$$m_\phi^2 = \frac{1}{3f_{RR}}(f_R - R f_{RR}) \simeq \frac{f_R}{3f_{RR}} \tag{65}$$
$$\Rightarrow \lambda_C \simeq \frac{2\pi}{m_\phi} \tag{66}$$

So we've required that in high-curvature regimes the mass of the scalaron becomes large, and it's Compton wavelength, limiting its propagation distance – this is our first early glance of a chameleon-like mechanism.

2.3 Conditions for Viable Models

$f(R) = R + \alpha R^2$ has been investigated since the 80s as an inflation model, but it's not good for late-time acceleration – we need something that predominantly modifies the low-R regime. Something like $f(R) = R - \alpha/R^n$ is a decent guess, but it fails to satisfy some necessary conditions for viable models. These are:

1. $f_R > 0$ for $R > R_0$, where R_0 is the scalar curvature today (note $R = 6/a^2(\dot{\mathcal{H}} + \mathcal{H}^2)$). Friedmann: $3\mathcal{H}^2 f_R = 8\pi G_N \rho \ldots$, don't want to change sign of G_{eff}.

2. $f_{RR} > 0$ for $R > R_0$ to ensure stability at high R, by having $m^2 > 0$ above. (Also for consistency with local tests.)

3. $f(R) \to R - 2\Lambda$ to preserve the early universe.

4. f_R small at current epochs, $\sim 10^{-5}$ or less – we will see that f_{R0} affects structure formation.

Some example models from the literature:

1. $f(R) = R - \mu R_c \left(\frac{R}{R_c}\right)^p$ with $0 < p < 1$ and μ, $R_c > 0$.

2. $f(R) = R - \mu R_c \frac{\left(\frac{R}{R_c}\right)^{2n}}{\left(\frac{R}{R_c}\right)^{2n} + 1}$ with n, μ, $R_c > 0$

3. $f(R) = R - \mu R_c \left[1 - \left(1 + \frac{R^2}{R_c^2}\right)^{-n}\right]$ with n, μ, $R_c > 0$

4. $f(R) = R - \mu R_c \tanh(R/R_c)$ with μ, $R_c > 0$

All of these models satisfy $f(R = 0) = 0$, so the effective CC vanishes in flat spacetime. They also all have a cosmological-constant-like regime for $R >> R_c$. For model 1, this requires that p be very close to 1, though. For model 4, $f(R) \to R - \mu R_c$ very quickly for $R >> R_c$.For models 2 and 3, in the $R >> R_c$ regime they are approximately:

$$f(R) = R - \mu R_c \left(1 - \left[\frac{R^2}{R_c^2}\right]^{-n}\right) \tag{67}$$

so $f(R) \to R - \mu R_c$ irregardless of n in this regime.

2.4 Cosmological Perturbations in $f(R)$

First we need the perturbed Einstein field equations. The 00 components of this is, where $\delta F = \delta f_R$ and we are briefly back in physical time:

$$\frac{k^2}{a^2}\Phi + 3H(\dot{\Phi} + H\Psi) = \frac{1}{2f_R}\left[3H\delta\dot{F} - \delta F\left(3\dot{H} + 3H^2 - \frac{k^2}{a^2}\right) - 3H\dot{f}_R\Psi - 3\dot{f}_R(\dot{\Phi} + H\Psi) - 8\pi G_N \rho_m \delta_m\right] \tag{68}$$

$$\Rightarrow -\frac{k^2}{a^2}\Phi = \frac{1}{2f_R}\left[8\pi G_N \rho_m \delta_m - \frac{k^2}{a^2}\delta F\right] \tag{69}$$

$$\Rightarrow \Phi = -\frac{1}{2f_R}\left[8\pi G_N \rho_m \frac{a^2}{k^2}\delta_m - \delta F\right] \tag{70}$$

where in the second line we've used the quasistatic approximation (QSA), that is, $|\dot{\Phi}| \sim \mathcal{H}|\Phi|$, $|\ddot{\Phi}| \sim \mathcal{H}^2|\Phi| << |k^2\Phi|$, and the same for Ψ, and even δF (see later). The first two parts of this are broadly equivalent to taking the Newtonian limit. The extension to the scalaron one might question, but it has been verified for a number of MG theories. See von Braun Bates, Noller & Ferreira for $f(R)$. Note that also $\dot{f}_R \sim H$.

Now the ij component is:

$$\Phi - \Psi = \frac{\delta F}{f_R} \tag{71}$$

$$\Rightarrow \Psi = -\frac{1}{2f_R}\left[8\pi G_N \rho_m \frac{a^2}{k^2}\delta_m + \delta F\right] \tag{72}$$

And the scalaron equation of motion, where we've dropped terms that are suppressed compared to $\delta\rho_m$ inside the Hubble radius, and used $\delta R = \delta F / f_{RR}$:

$$\delta\ddot{F} + 3H\delta\dot{F} + \delta F\left(\frac{k^2}{a^2} + m^2\right) = \frac{8\pi G_N}{3}\rho_m \delta_m \tag{73}$$

where recall $m^2 = f_R/(3f_{RR})$.

Now for the fluid equations. Life is often easiest if we express our time derivatives in terms of the variable $x = \ln a$. We'll denote these derivs as $f' = df/dx = \mathcal{H}\dot{f}$. Then eqs.35 and 34, where $\tilde{\theta} = \theta/\mathcal{H}$:

$$\delta' + \tilde{\theta} - 3\Phi' = 0 \tag{74}$$

$$\tilde{\theta}' + \left(1 + \frac{\mathcal{H}'}{\mathcal{H}}\right)\tilde{\theta} - \frac{k^2}{\mathcal{H}^2}\Psi = 0 \tag{75}$$

where we've used $\dot{\theta} = \dot{H}\tilde{\theta} + H\dot{\tilde{\theta}} = \mathcal{H}^2(\tilde{\theta}' + \tilde{\theta}\mathcal{H}'/\mathcal{H})$ and divided the expression through by $\mathcal{H}^2$. Now take a derivative of the delta eq and sub in the theta eq:

$$\delta'' = 3\Phi'' - \tilde{\theta}' \tag{76}$$

$$= 3\Phi'' + \left(1 + \frac{\mathcal{H}'}{\mathcal{H}}\right)\tilde{\theta} - \frac{k^2}{\mathcal{H}^2}\Psi \tag{77}$$

$$= 3\Phi'' - \left(1 + \frac{\mathcal{H}'}{\mathcal{H}}\right)\delta' - 3\left(1 + \frac{\mathcal{H}'}{\mathcal{H}}\right)\Phi' - \frac{k^2}{\mathcal{H}^2}\Psi \tag{78}$$

$$\Rightarrow \quad \delta'' + \left(1 + \frac{\mathcal{H}'}{\mathcal{H}}\right)\delta' + \frac{k^2}{\mathcal{H}^2}\Psi \simeq 0 \tag{79}$$

Now, in the subhorizon regime, $k^2 >> m^2$ in eq.(73). Further applying the QSA we find:

$$\delta F \simeq \frac{1}{3} \cdot 8\pi G_N \rho_m \frac{a^2}{k^2}\delta_m \tag{80}$$

Now use eq.(72)

$$\Psi = -\frac{1}{2f_R} \cdot \frac{4}{3} \cdot \left[8\pi G_N \rho_m \frac{a^2}{k^2}\delta_m\right] \tag{81}$$

$$\Phi = -\frac{1}{2f_R} \cdot \frac{2}{3} \cdot \left[8\pi G_N \rho_m \frac{a^2}{k^2}\delta_m\right] \tag{82}$$

Then finally eq.79 becomes:

$$\delta'' + \left(1 + \frac{\mathcal{H}'}{\mathcal{H}}\right)\delta' - \frac{1}{2f_R\mathcal{H}^2} \cdot \frac{4}{3} \cdot \left[8\pi G_N \rho_m a^2 \delta\right] = 0 \tag{83}$$

$$\delta'' + \left(1 + \frac{\mathcal{H}'}{\mathcal{H}}\right)\delta' - \frac{4}{3} \cdot \frac{3}{2}\Omega_m \delta = 0 \tag{84}$$

where we've used

$$\Omega_m = \frac{8\pi G_N a^2 \rho_m}{3\mathcal{H}^2 \cdot f_R} \tag{85}$$

The factor of f_R in the denominator here is not present in LCDM. Now, we see that this differs by a factor of 4/3 compared to our usual LCDM growth equation.

Let's solve this equation in the matter-dominated era again. Here $\Omega_m = 1$, and $\mathcal{H}'/\mathcal{H} = -1/2$. So, trying the usual $\delta \propto e^{px}$:

$$\delta'' + \frac{1}{2}\delta' - 2\delta = 0 \tag{86}$$

$$\Rightarrow 2p^2 + p - 4 = 0 \tag{87}$$

$$\Rightarrow p = \frac{1}{4}\left(-1 \pm \sqrt{33}\right) \tag{88}$$

Taking the growing mode, we have $\delta \propto a^{(\sqrt{33}-1)/4}$, and hence:

$$f = \frac{d\ln\delta}{d\ln a} = \frac{\sqrt{33}-1}{4} \simeq 1.186 \tag{89}$$

Growth is enhanced w.r.t. LCDM! Here we solved the e.o.m. of the scalaron in a sub-horizon regime, which resulted in this $k-$independent result again. The full equation can be solved numerically.

2.5 DGP Gravity

$$S_{DGP} = \frac{1}{2\kappa_{5D}^2}\int d^5X\sqrt{-\tilde{g}}\tilde{R} + \frac{1}{2\kappa_{4D}^2}\int d^4x\sqrt{-g}R - \int d^5X\sqrt{-\tilde{g}}\mathcal{L}_m \tag{90}$$

where $\tilde{g}_{AB}$ is the metric in the 5D bulk and $g_{\mu\nu} = \partial_\mu X^A \partial_\nu X^B \tilde{g}_{AB}$ is the induced metric on the 3-brane. The effective Plancks constants of bulk and brane are here denoted

$$\kappa_{5D}^2 = M_{5D}^{-3} \qquad \kappa_{4D}^2 = M_{4D}^{-2} \equiv 8\pi G_N \tag{91}$$

$$ds^2 = -n^2(\tau, y)d\tau^2 + a^2(\tau, y)\gamma_{ij}dx^i dx^j + dy^2 \tag{92}$$

$$\tilde{G}_{AB} = \tilde{R}_{AB} - \frac{1}{2}\tilde{R}\tilde{g}_{\mu\nu} = \kappa_{5D}^2 \tilde{T}_{AB} \tag{93}$$

DGP considers the bulk to be Minkowski, so there can be no matter in the bulk. The brane can contribute a possible tension to the stress-energy tensor *on* the 3-brane, but here we choose this tension to be zero. However, the brane itself has a scalar curvature contribution (depends on a and n and derivatives, too messy to write out, diagonal):

$$\tilde{T}_{AB} = T_{AB}^m + U_{AB} \tag{94}$$

$$T_{AB} = (-\rho_M, P_m, P_m, P_m, 0) \tag{95}$$

After thinking carefully about junction conditions at the brane (continuity of a'' and n''), we obtain the following Friedmann equation:

$$\frac{\epsilon}{r_c}\sqrt{H^2 + \frac{K}{a^2}} = H^2 + \frac{K}{a^2} - \frac{\kappa_{4D}^2}{3}\rho_m \tag{96}$$

$$\text{Crossover scale} \qquad r_c = \frac{\kappa_{5D}^2}{2\kappa_{4D}^2} = \frac{M_{4D}^2}{2M_{5D}^3} \tag{97}$$

$$H = \frac{\dot{a}_b}{a_b n_b} \tag{98}$$

where subscript b indicates evaluation on the brane. ϵ has come from taking a square root, and gives to branches to the theory for $\epsilon = \pm 1$. Setting the curvature $K = 0$:

$$H^2 - \frac{\epsilon}{r_c}H = \frac{\kappa_{4D}^2}{3}\rho_m \tag{99}$$

and matter evolves according to it's usual equation w.r.t. to the time $dt = n_b d\tau$. Because of this $\rho_m \propto a^{-3}$ as usual, and hence in the far future we reach the dS solution for the case $\epsilon = +1$:

$$H \to H_{dS} = \frac{1}{r_c} \tag{100}$$

where r_c needs to be of order $\sim H_0^{-1}$. For this reason this branch of the model is often known as the self-accelerating branch. The acceleration is the result of gravitational 'leakage' into the bulk on horizon scales.

A flat DGP model is consistent with SN data, but comes under pressure from BAO and the CMB shift parameter; an open model fits slightly better. See Maartens and Majerotto 2006 (where $\Omega_{r_c}^0 = ([1 - \Omega_m^0]/2)^2$).

2.6 DGP Perturbations

Perturbed line element:

$$ds^2 = -(1 + 2\Psi)n(t, y)^2 d\tau^2 + (1 - 2\Phi)\, A(\tau, y)^2 \delta_{ij}dx^i dx^j + 2r_c \partial_i B dx^i dy + (1 + 2E)dy^2 \tag{101}$$

This time we've jumped straight to a flat 3-metric, $\gamma_{ij} = \delta_{ij}$, and we've generalised to A; the solution for the self-accelerating branch is $A = a(\tau)(1 + Hy)$, which reduces to a at $y = 0$ on the brane. B is identified as a brane-bending mode.

We're only going to need one part of the full 5D Einstein equations; the component $\delta\tilde{G}_5^5 = 0$ gives:

$$\frac{\nabla^2}{A^2}(\Psi - 2\Phi) - \frac{r_c}{A^2}\left(\frac{2A'}{A} + \frac{n'}{n}\right)\nabla^2 B = 0 \tag{102}$$

In this case, we get to the expressions we want more directly by using the junction conditions at the brane. These are:

$$K_{\mu\nu} - Kg_{\mu\nu} = -\frac{\kappa_{5D}^2}{2}T_{\mu\nu} + r_c G_{\mu\nu} \tag{103}$$

$$h_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu \qquad K_{\mu\nu} = h_\mu^\lambda \nabla_\lambda n_\nu \tag{104}$$

The 00, spatial and 55 components of eq.(103) give:

$$-\frac{2k^2}{a^2}\Phi = \kappa_{4D}^2\delta\rho_m + \frac{k^2}{a^2}B + \frac{3}{r_c}\Phi' \tag{105}$$

$$\Psi - \Phi = B \tag{106}$$

$$\Psi' - 2\Phi' = 0 \tag{107}$$

Now, $\Phi' \sim (k/a)\Phi$ (note that prime is a *spatial* derivative here). Comparing the last two terms of the 00 equation:

$$\frac{k^2}{a^2}\left(B + \frac{3}{r_c}\frac{a}{k}\Phi\right) \tag{108}$$

If the mode wavelength $\lambda \sim k^{-1} << r_c$, this term is small, and we drop it. Using eqs.103, 105 and106:

$$-\frac{2k^2}{a^2}\Phi = \kappa_{4D}^2\delta\rho_m + \frac{k^2}{a^2}B \qquad -\frac{2k^2}{a^2}\Psi = \kappa_{4D}^2\delta\rho_m - \frac{k^2}{a^2}B \tag{109}$$

Now from eq.(102) we get, using the slip relation:

$$\Psi - 2\Phi - r_c\left(\frac{2A'}{A} + \frac{n'}{n}\right)B = B\left[1 - r_c\left(\frac{2A'}{A} + \frac{n'}{n}\right)\right] - \Phi = 0 \tag{110}$$

Subbing this into 109:

$$-\frac{2k^2}{a^2}\Phi = \kappa_{4D}^2\delta\rho_m + \frac{k^2}{a^2}\Phi\frac{1}{[\ldots]} \quad \Rightarrow \quad -\frac{2k^2}{a^2}\Phi\left(1 + \frac{1}{2[\ldots]}\right) = \kappa_{4D}^2\delta\rho_m \tag{111}$$

$$-\frac{2k^2}{a^2}\Phi = \kappa_{4D}^2\delta\rho_m \times \frac{2[\ldots]}{1+2[\ldots]} \quad \Rightarrow \quad -\frac{2k^2}{a^2}\Phi = \kappa_{4D}^2\delta\rho_m \times \left(1 - \frac{1}{1+2[\ldots]}\right) \tag{112}$$

$$3\beta = 1 + 2\left[1 - r_c\left(\frac{2A'}{A} + \frac{n'}{n}\right)\right] = 3 - 2r_c\left(\frac{2A'}{A} + \frac{n'}{n}\right) \tag{113}$$

So finally we have:

$$-\frac{2k^2}{a^2}\Phi = \kappa_{4D}^2\delta\rho_m \times \left(1 - \frac{1}{3\beta}\right) \qquad -\frac{2k^2}{a^2}\Psi = \kappa_{4D}^2\delta\rho_m \times \left(1 + \frac{1}{3\beta}\right) \tag{114}$$

Then recall eq.79:

$$\delta'' + \left(1 + \frac{\mathcal{H}'}{\mathcal{H}}\right)\delta' + \frac{k^2}{\mathcal{H}^2}\Psi \simeq 0 \tag{115}$$

$$\Rightarrow \delta'' + \left(1 + \frac{\mathcal{H}'}{\mathcal{H}}\right)\delta' - \frac{3}{2}\Omega_{M0}\delta\left(1 + \frac{1}{3\beta}\right) = 0 \tag{116}$$

where this time we have kept Ω_M in. Using $A = a(\tau)(1 + Hy)$ and $n = 1 + H(1 + \dot{H}/H^2)y$, one can easily show that (exercise – remember the brane is at $y = 0$):

$$\beta = 1 - 2Hr_c\left(1 + \frac{\dot{H}}{3H^2}\right) \tag{117}$$

Deep in the matter era, the horizon is well below the crossover scale, so $Hr_c >> 1$ and $\beta \simeq -Hr_c$, i.e. large and negative (recall $\dot{H} = -0.5H^2$ in the matter era). During this phrase, the growth equation is close to the GR/LCDM one.

In contrast, in the late dS solution $H \to H_{dS}$, a constant, so $\beta \to 1 - 2Hr_c \simeq -1$ if r_c is of order the horizon. Then $1 + 1/3\beta \simeq 2/3$, modifying the growth index again. This late-time case is harder to solve by hand, but we can see that the driving term of the equation is weakened w.r.t. LCDM (it is $1.5\Omega_M$ in GR, and Ω_M here).

We saw that in the matter era $f = 1$; at late times it's evolution is often parameterised as $f \simeq \Omega_M^\gamma$, where $\gamma = 0.55$ in GR, and $\gamma \simeq 0.68$ in DGP. Reference: Linder, PHYSICAL REVIEW D 72, 043529 (2005) Cosmic growth history and expansion history.

We can see from eq.(114):

$$G_{\text{eff}} = G_N \times \left(1 + \frac{1}{3\beta}\right) \tag{118}$$

Comparing this to JBD, where

$$G_{\text{eff}} = G_N \times \frac{4 + 2\omega_{JBD}}{3 + 2\omega_{JBD}} \tag{119}$$

we find $\omega_{JBD} \equiv \frac{3}{2}(\beta - 1)$. But there's a deep sickness here. Since for the accelerating branch of DGP β is negative, this corresponds to $\omega_{JBD} < -\frac{3}{2}$. Recall that ω_{JBD} parameterises the kinetic term of the scalar. We find that the scalar degree of freedom in DGP is a ghost, meaning its KE is negative in the Einstein frame. This essentially makes the theory inviable. Note that this is *not* true of the non-accelerating branch, sign the sign in eq.(117) is opposite.

2.7 Horndeski Gravity & Galileons

The study of the scalar d.o.f. in DGP gave rise to another family of theories. Horndeski gravity (originally Walter Horndeski 1974) is the most general theory one can write down involving one additional scalar d.o.f. added to a metric theory of gravity in 4D, where the eoms are up to second order in derivatives. As such, it encompasses nearly all of the scalar-tensor type models: quintessence & JBD, k-essence, phantom DE, $f(R)$, Galileons, etc. What takes it beyond regular ST is the presence of derivative couplings.

$$S_{HD} = \int d^4x\sqrt{-g}\,[\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 - \mathcal{L}_m(\psi, g_{\mu\nu})] \tag{120}$$

$$\mathcal{L}_2 = G_2(\phi, X) \tag{121}$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi \tag{122}$$

$$\mathcal{L}_4 = G_4(\phi, X)R + \frac{d}{dX}G_4(\phi, X)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)\right] \tag{123}$$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \tag{124}$$

$$-\frac{1}{6}\frac{d}{dX}G_5(\phi, X)\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) + 2(\nabla_\mu\nabla^\nu\phi)(\nabla_\nu\nabla^\rho\phi)(\nabla_\rho\nabla^\mu\phi)\right] \tag{125}$$

where $X = -\nabla_\mu\phi\nabla^\mu\phi/2 \equiv \dot{\phi}^2/2$ on a Minkowski (or FRW phys time) background. E.g. $f(R)$ is recovered for $G_4 = \phi = f_R$, $G_2 = V(\phi) = f(R) - Rf_R$, $G_3 = 0$.

Another interesting case of study are the Galileon family of theories:

$$G_2 = -c_2X \qquad G_2 = \frac{2c_3}{M^3}X \qquad G_4 = -\left[\frac{M_P^2}{2} + \frac{c_4}{M^6}\right] \qquad G_5 = \frac{3c_5}{M^9}X^2 \tag{126}$$

where the c_i are constants and M is a mass. Here, the G_i only depend on X, never ϕ directly. This means that the Lagrangian is invariant under the shift $\partial_\mu\phi \to \partial_\mu\phi + b_\mu$. The connection of this symmetry to cosmology isn't immediately obvious, but it's interesting from a theory viewpoint: it ensures that the classical solutions of the theory are safe from corrections at loop order. i.e. a high-energy completion of the theory won't invalidate this low-energy treatment.

2.8 Viability Bounds on Horndeski

The recent results from the BNS were very important for constraining sectors of general HD theory. To see why, we first need to calculate the GW speed in this framework. To do this, we will write the line element in the ADM form (different to before):

$$ds^2 = -N^2dt^2 + a^2(t)\gamma_{ij}dx^idx^j \tag{127}$$

We can set $\bar{N} = 1$ by our choice of time coordinate. Now perturb $N = 1 + \delta N$, $N_i = \partial_i\psi$ (not needed) and

$$\gamma_{ij} = a^2e^{2\zeta}(e^h)_{ij} = \delta_{ij} + h_{ij} + \frac{1}{2}h_i^kh_{kj} + \ldots \tag{128}$$

where h_{ij} satisfies the usual transverse traceless conditions, $\partial^ih_{ij} = h_i^i = 0$, and a spatial gauge transf has been used. Then the tensor part of the second-order action:

$$S_{HD}^{(T)(2)} = \frac{1}{8}\int dtd^3x\,a^3\left[\mathcal{G}_T\,\dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}\,(\partial_kh_{ij})^2\right] \tag{129}$$

$$\mathcal{G}_T = 2\left[G_4 - 2XG_{4X} - X\left(H\dot{\phi}G_{5X} - G_{5\phi}\right)\right] \tag{130}$$

$$\mathcal{F}_T = 2\left[G_4 - X\left(\ddot{\phi}G_{5X} + G_{5\phi}\right)\right] \tag{131}$$

$$\Rightarrow c_{GW}^2 = \frac{\mathcal{F}_T}{\mathcal{G}_T} \tag{132}$$

One requires $c_{GW}^2 > 0$ and $c_S^2 > 0$ to prevent the perturbation modes from exhibiting exponential growth, i.e. **gradient instabilities**. Likewise we require $\mathcal{G}_T > 0$ (and similarly for scalar modes) to guarantee the kinetic terms of scalar and tensor perturbations are positive, i.e. absence of **ghost instabilities.**

The comparison of gamma-ray spectrograph and GW detector data from event GW170817 tells us that the EM and GW counterparts of this event reached Earth within two seconds of oneanother. Let's work out the implications of this, pre-emptively using the deviation α_T; note that for a very local source we can use Euclidean geometry:

$$c_{GW}^2 = 1 + \alpha_T(z) \tag{133}$$

$$\Delta t = t_{GW} - t_{EM} = \frac{d}{c_{GW}} - \frac{d}{c} \tag{134}$$

$$= \frac{d}{c}\left(\frac{1}{\sqrt{1+\alpha_T(0)}} - 1\right) \simeq \frac{d}{c}\frac{\alpha_T}{2} \tag{135}$$

$$\Rightarrow \alpha_T(0) = \frac{2\Delta t c}{d} = \frac{4s \cdot 3\times 10^8 \mathrm{m\,s^{-1}}}{40\,\mathrm{Mpc}} \simeq 10^{-15} \tag{136}$$

This is tiny! Note that because the same ingredients appear in $\mathcal{F}_T$ and $\mathcal{G}_T$, we've no easy way to make the denominator parameterically larger than the numerator. We could look at making the numerator very small but this is hard to achieve as a function of redshift due to the presence of things like $\ddot{\phi}$ and X. (Loophole here w.r.t. current data only).

Barring a finely-tuned situation, our most likely alternative seems to be that $G_{4X} = G_{5X} = G_{5\phi} = 0$. But note that now G_5 is at most a constant, and under and integration by parts we reach $\nabla^\mu G_{\mu\nu} = 0$, so $\mathcal{L}_5 = 0$. We lose all of the quintic Lagrangian, and much of the quartic. Looking back at our Galileon example, we must have $c_4 = c_5 = 0$. Only the cubic term is left, though this is constrained in other ways (see later).

2.9 The Alpha Parameters

As you can guess from the action, Horndeski can be messy to work with. At the level of linear PT, we define another set of parameters that actually appear in the field equations. Note that we can pull $\mathcal{G}_T$ out of the action, where it acts like an effective (possible time-evolving Planck mass):

$$M_*^2 = \mathcal{G}_T \qquad \alpha_M = \frac{1}{H}\frac{d\ln M_*^2}{dt} \tag{137}$$

$$H^2 M_*^2 \alpha_K = 2X\left(G_{2X} + 2XG_{2XX} - 2G_{3\phi} - 2XG_{3\phi X}\right) + 12\dot{\phi}XH\left(G_{3X} + XG_{3XX}\right) \tag{138}$$

$$H^2 M_*^2 \alpha_B = 2\dot{\phi}\left(XG_{3X} - G_{4\phi}\right) \tag{139}$$

Explain these. NB: these are the reduced versions, ie. post-GW170817, where we've applied

$$\alpha_T = 2X\left(2G_{4X} - 2G_{5\phi} - (\ddot{\phi} - \dot{\phi}H)G_{5X}\right) = 0 \tag{140}$$

Reference: Bellini & Sawicki 2014, show table. These parameters have grown in popularity because they i) have physical meaning, ii) appear more directly in the common linear equations, e.g. the Poisson equation:

$$2\nabla^2\Phi = 8\pi G_{\rm eff}\rho a^2\delta \tag{141}$$

$$\frac{G\mathrm{eff}}{G_N} = 1 + \frac{\alpha_B(\alpha_B + 2\alpha_M)}{(\alpha_K + 3\alpha_B{}^2)c_s^2} \tag{142}$$

We can now express the stability constraints on the scalar sector more easily. The equivalent of eq.129 is

$$S_{HD}^{(S)(2)} = \frac{1}{8}\int dt d^3x\, a^3\left[\mathcal{G}_S\,\dot{\zeta}^2 - \frac{\mathcal{F}_S}{a^2}(\partial\zeta)^2\right] \tag{143}$$

We can now see that we require the following:

$$\mathcal{G}_S > 0 \qquad \Rightarrow \qquad \frac{2M_*^2(\alpha_K + 3\alpha_B{}^2)}{(2-\alpha_B)^2} > 0 \tag{144}$$

$$c_s^2 = \frac{\mathcal{F}_T}{\mathcal{G}_T} = -\frac{1}{H^2(\alpha_K + 3\alpha_B{}^2)}\left[(2-\alpha_B)\left(\dot{H} - H^2\alpha_M - \frac{1}{2}H^2\alpha_B\right) - H\dot{\alpha_B} + \frac{\rho_m + P_m}{M_*^2}\right] > 0 \tag{145}$$

Note that the combination appearing in the denominator of 144 has no dependence on α_K, so we don't expect this to affect galaxy clustering.

Note that since the G_i are functions, the α_i are also functions. However, if we don't specify a full model, we can't calculate the evolution of ϕ and X, and hence the time dependence of α_i. The standard approach to this is to use an ansatz, the most common ones being:

$$\alpha_i = c_i a \qquad \alpha_i = c_i a^p \qquad \alpha_i = c_i \Omega_\Lambda(a) \tag{146}$$

The ability of these to represent 'real' models has been called into question (Linder). Nevertheless, using such ansatzes, the α_i can be constrained with data $\to$ Noller plot. We see (note no α_K, talk about codes later):

- CMB data rules out things too far from the line $\alpha_M \propto \alpha_B$, due to the ISW effect. However, large values of both are allowed.
- Large negative values of both are ruled out by stability constraints.
- The growth of LSS cuts out higher values of α_M.
- Things are tighter for the $\propto a$ ansatz, which kicks in at much earlier times (consider $z = 2$).

The smallest contours and solid line represent a recent theoretical bound claiming that $|\alpha_M + \alpha_B| < 10^{-2}$ (technical result). Reference: Creminelli 2019.

At one time there was a fifth parameter, α_H, which quantified the disformal symmetries of a theory. However, it was shown (Creminelli 2018) that in these theories GW can decay into the DE scalar. The cross-section is $\Gamma_{\gamma\to\pi\pi} \propto (c_s^2 - 1)$, so there is a loophole here if the sound speed is 1.

Finally, we should note that there is a caveat to the GW results. The mass scale that appears in the Galileons can be given a ballpark estimate:

$$M \lesssim \Lambda_{HD} \sim (M_P H_0^2)^{\frac{1}{3}} \sim 260\text{Hz} \tag{147}$$

where Λ_{HD} is the cut-off scale of Horndeski, which we think of as an EFT for scalar degrees of freedom. Note that the value of this cut-off is right in the middle of the LIGO band for GW170817. Therefore, it is just possible that we are using the EFT outside of its regime of validity, and that really we need the full UV theory to compute what we expect to see at events like GW170817. Reference: de Rham & Melville 2018.

3 Screening Mechanisms

Need to satisfy SS tests, e.g. Several types: chameleon, dilation, symmetron, Vainshtein, K-mouflage. Chameleon and Vainshtein are the dominant representatives.

3.1 Chameleon Mechanism

Start from an ST action in the Einstein frame, similar to eq.51:

$$S = \int d^4x\sqrt{-g}\left\{\frac{M_P^2}{2}R - \frac{1}{2}\nabla_\mu\varphi\nabla^\mu\varphi - V(\phi)\right\} + \mathcal{S}_M(\psi, A^2(\phi)g_{\mu\nu}) \tag{148}$$

Take ϕ equation, where $\tilde{g}_{\mu\nu} = A^2(\phi)g_{\mu\nu}$ is the Jordan frame metric:

$$\frac{\delta S}{\delta\phi} = \sqrt{-g}\left[\Box\phi - \partial_\phi V\right] + \frac{\delta S}{\delta\tilde{g}_{\mu\nu}}\frac{\delta\tilde{g}_{\mu\nu}}{\delta\phi} = 0 \tag{149}$$

$$\Rightarrow \quad \sqrt{-g}\left[\Box\phi - \partial_\phi V\right] + \frac{\sqrt{-\tilde{g}}}{2}\tilde{T}^{\mu\nu}g_{\mu\nu}\cdot 2A\partial_\phi A = 0 \tag{150}$$

$$\Rightarrow \quad \sqrt{-g}\left[\Box\phi - \partial_\phi V\right] + \sqrt{-\tilde{g}}\tilde{T}\cdot\frac{\partial_\phi A}{A} = 0 \tag{151}$$

$$\Rightarrow \quad \Box\phi = \partial_\phi V - A^3\tilde{T}\partial_\phi A \tag{152}$$

where we've used $\sqrt{-\tilde{g}} = A^4\sqrt{-g}$. Now using $A^4\tilde{T} = T$ and $T = -A\rho_m$:

$$\Box\phi = \partial_\phi V_{\text{eff}} \qquad\qquad V_{\text{eff}} = V + \rho_m A(\phi) \tag{153}$$

Consider how this potential varies for the simple choices $V \propto \phi^{-n}$, $A = \phi \rightarrow$ sketches. Note that this ties back into the idea of a density-dependent mass, as we saw earlier for $f(R)$ (which ofc is equivalent to ST):

$$m^2 = \partial_\phi^2 V_{\text{eff}} = \partial_\phi^2 V + \rho_m\partial_\phi^2 A(\phi) \tag{154}$$

Two common forms used are a Ratra-Peebles potential, and a linear function for A (since the field excursion must be smaller than the Planck mass):

$$V(\phi) = \frac{M^{4+n}}{\phi^n} \qquad\qquad A(\phi) \simeq 1 + \xi\frac{\phi}{M_P} \tag{155}$$

To study how this works in more detail, let's consider the simple example of a spherical, uniform density object embedded in a flat background:

$$\rho = \rho_a, \quad r > R \qquad\qquad \rho = \rho_o, \quad r < R \tag{156}$$

Eq.153 becomes

$$\phi'' + \frac{2}{r}\phi' = V_{,\phi} + \xi\frac{\rho}{M_P} \tag{157}$$

A full solution for general V would require numerics, but we can infer basic features more heuristically. First, we posit that deep inside the object where screening is complete, ϕ is very massive and cannot evolve – hence it must adopt a constant value, $\phi = \phi_o$. At large distances it must also tend to a constant $\phi \rightarrow \phi_{amb}$. Then there must be a transition regimes, within a Compton wavelength $r\ m^{-1}$ where the solution evolves like $1/r$. This leads us to guess the form:

$$\phi = B + \frac{A}{r} \tag{158}$$

$$\phi = \phi_a + \frac{R}{r}(\phi_o - \phi_a) \tag{159}$$

where the last line comes from requiring $\phi = \phi_o$ at $r = R$. This solution solves the Laplace equation, $\nabla^2\phi \simeq 0$. On these grounds we make an electrostatic analogy where any scalar 'charge' in the body is confined to a thin shell of thickness ΔR around its outside. This charge density $\xi\rho\Delta R/M_P$ supports a field gradient discontinuity:

$$\frac{d\phi}{dr}\Big|_{r=R_+} = \xi\frac{\rho}{M_P}\Delta R \equiv -\frac{1}{R}(\phi_o - \phi_a) \tag{160}$$

Second equality comes from eq.(159). Now we use

$$|\Phi|\Big|_R = \frac{GM}{R} = \frac{M}{8\pi M_P^2 R} = \frac{\rho R^2}{6M_P^2} \tag{161}$$

$$\Rightarrow \Delta R = -\frac{1}{R}(\phi_o - \phi_a) \cdot \frac{1}{\xi} \cdot \frac{R^2}{6M_P|\Phi|_R} \tag{162}$$

$$\Rightarrow \frac{\Delta R}{R} = \frac{(\phi_a - \phi_o)}{6M_P\xi|\Phi|_R} \tag{163}$$

The field profile outside the object can then be written:

$$\phi(r > R) = \phi_a - \frac{\Delta R}{R}\frac{6M_P\xi|\Phi|_R}{r} \tag{164}$$

$$= \phi_a - \frac{\Delta R}{R}\frac{\xi}{r}\frac{6M}{8\pi M_P R} \tag{165}$$

$$= \phi_a - \frac{3\xi}{4\pi M_P}\frac{\Delta R}{R}\frac{Me^{-m_a(r-R)}}{r} \qquad \text{(screened)} \tag{166}$$

where the last line the Yukawa factor for a massive field has been fudged in. This profile is like that of a massive scalar, except for the coupling has been reduced by the factor $\Delta R/R << 1$, the 'thin shell' factor.

This calculation breaks down if the RHS is too big, such that $\Delta R \gtrsim R$. This happens if $|\Phi|$ is small, and then the object is completely unscreened. The field profile outside the object can then be written:

$$\phi(r > R) = \phi_a - \frac{3\xi}{4\pi M_P}\frac{Me^{-m_a(r-R)}}{r} \qquad \text{(unscreened)} \tag{167}$$

In words: in a screened case, the scalar only couples to the thin shell regime, as it is too massive to fluctuate deeper inside the body. The MG fifth force of the theory is then hugely suppressed compared to the regular gravitational attraction of the rest of the body, yielding predictions effectively the same as in GR.

Show fig. 6 of Joyce?

3.2 Vainshtein Screening

Let's consider this in the example of the cubic Galileon, which corresponds to eqs.(126) with $c_2 = -6$, $c_3 = -1$:

$$\mathcal{L} = -2(\nabla\phi)^2 - \frac{1}{\Lambda^3}\Box\phi(\nabla\phi)^2 + \frac{g\phi}{M_P}T \tag{168}$$

g here controls the coupling to matter, which for convenience here we will take to be a (non-relativistic) point source of mass M. Then $T = -M\delta^{(3)}(x)$. The equation of motion for ϕ is:

$$\delta\phi\left[6\Box\phi + \frac{2}{\Lambda^3}\left((\Box\phi)^2 + \nabla_\mu(\Box\phi)\nabla^\mu\phi - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) - \nabla_\mu\phi\Box(\nabla^\mu\phi)\right) - \frac{gM}{M_P}\right] = 0 \tag{169}$$

$$6\Box\phi + \frac{2}{\Lambda^3}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)\right] = \frac{gM}{M_P}\delta^{(3)}(x) \tag{170}$$

$$\nabla\cdot\left[6\nabla\phi + \hat{r}\frac{4}{\Lambda^3}\frac{(\nabla\phi)^2}{r}\right] = \frac{gM}{M_P}\delta^{(3)}(x) \tag{171}$$

Now we integrate the LHS using the divergence rule, recall $\int(\nabla\cdot F)dV = \int(F\cdot\hat{n})dS$, and divide the dS through:

$$6\phi' + \frac{4}{\Lambda^3}\frac{(\phi')^2}{r} = \frac{gM}{4\pi r^2 M_P} \tag{172}$$

This is an algebraic equation for ϕ':

$$(\phi')^2 + \frac{3\Lambda^3 r}{2}\phi' - \frac{\Lambda^3}{4}\frac{gM}{4\pi r M_P} = 0 \tag{173}$$

$$\phi' = \frac{1}{2}\left\{\frac{3\Lambda^3 r}{2} \pm \sqrt{\frac{9\Lambda^6 r^2}{4} + \Lambda^3 \frac{gM}{4\pi r M_P}}\right\} \tag{174}$$

$$= \frac{3\Lambda^3 r}{4}\left\{1 \pm \sqrt{1 + \frac{gM}{9\pi r^3 M_P \Lambda^3}}\right\} \tag{175}$$

$$= \frac{3\Lambda^3 r}{4}\left\{1 + \sqrt{1 + \frac{1}{9\pi}\left(\frac{r_V}{r}\right)^3}\right\} \tag{176}$$

$$\Rightarrow r_V = \frac{1}{\Lambda}\left(\frac{gM}{M_P}\right)^{\frac{1}{3}} \tag{177}$$

where we pick the + solution such that $\phi' \to 0$ at large distances from the source (NB: the other sign matches up to the ghostly self-accelerating solution of DGP). Look at the limits here, first $r >> r_V$:

$$\phi' \to \frac{3\Lambda^3 r}{4}\frac{1}{18\pi}\left(\frac{r_V}{r}\right)^3 = \frac{\Lambda^3}{24\pi}\frac{r_V^3}{r^2} = \frac{g}{3}\frac{M}{8\pi M_P r^2} \tag{178}$$

Note that the second part above is the usual gravitational force on the body. So

$$\frac{F_\phi}{F_g}\Big|_{r>>r_V} \simeq \frac{g^2}{3} \tag{179}$$

In DGP recovers the usual factor of 1/3. Now look at the limits here, first $r << r_V$:

$$\phi' \to \frac{3\Lambda^3 r}{4}\frac{1}{\sqrt{9\pi}}\left(\frac{r_V}{r}\right)^{\frac{3}{2}} = \frac{\Lambda^3}{4\sqrt{\pi}}\frac{r_V^{\frac{3}{2}}}{\sqrt{r}} = \frac{\Lambda^3}{4\sqrt{\pi}}\frac{r_V^3}{\sqrt{r r_V^3}} \tag{180}$$

$$\simeq \frac{r^{\frac{3}{2}}}{r_V^{\frac{3}{2}}}\frac{gM}{M_P r^2} \tag{181}$$

$$\Rightarrow \frac{F_\phi}{F_g}\Big|_{r<<r_V} \simeq \left(\frac{r}{r_V}\right)^{\frac{3}{2}} \tag{182}$$

i.e. inside the Vainshtein radius, the fifth force is highly suppressed – the screening mechanism. Note that unlike the chameleon, there was no environmental dependence on inside and outside the body; only the total mass matters. Plugging in the numbers, one finds that the Vainshtein radius of the Sun is a few kpc.
→ Show Barreira plot of NFW haloes, largely independent of M. R_{200} is the radius which the density drops below 200 times the critical matter density at a given redshift.

4 More on Observational Signatures of MG

This section is necessarily more qualitative.

4.1 Other Parameterised Approaches

- We met Horndeski, a kind of parameterisation. Another, simpler and more phenomenological parameterisation predates this. We have seen already several modified Poisson equations:

$$-2\frac{k^2}{a^2}\Psi = \frac{1}{f_R}\cdot\frac{4}{3}\cdot[8\pi G_N\rho_m\delta_m] \tag{183}$$

$$-\frac{2k^2}{a^2}\Psi = \kappa_{4D}^2\delta\rho_m\times\left(1+\frac{1}{3\beta}\right) \tag{184}$$

$$2\nabla^2\Phi = 8\pi G_{\rm eff}\rho a^2\delta \qquad \frac{G{\rm eff}}{G_N} = 1+\frac{\alpha_B(\alpha_B+2\alpha_M)}{(\alpha_K+3\alpha_B{}^2)c_s^2} \tag{185}$$

$$\Rightarrow 2\nabla^2\Psi = 8\pi G_N\rho_m\delta_m\,[1+\mu(a,k)] \tag{186}$$

Where in the last line the general function μ depends on redshift, and possibly on k. Then recall eq.79:

$$\delta'' + \left(1+\frac{\mathcal{H}'}{\mathcal{H}}\right)\delta' + \frac{k^2}{\mathcal{H}^2}\Psi \simeq 0 \tag{187}$$

$$\Rightarrow \delta'' + \left(1+\frac{\mathcal{H}'}{\mathcal{H}}\right)\delta' - \frac{3}{2}\Omega_M(x)\,[1+\mu(x,k)] = 0 \tag{188}$$

We could solve this to get the modified growth rate in a general scenario. Conversely, by constraining the growth rate one can constrain general forms of μ. Another key function appears in the lensing convergence power spectrum, which goes like $\kappa\sim\int\partial^2(\Phi+\Psi)$.

$$\Phi+\Psi = [1+\Sigma(x,k)](\Phi+\Psi)_{GR} \tag{189}$$

So a non-zero slip relation gives a non-zero Σ. Sometimes the slip itself is parameterised as $\gamma=\Phi/\Psi$, but μ and Σ are closest to observables.

- It's be reasoned (Silvestri, Pogosian, Buniy) that a sensible form for the scale-dependence is

$$\mu(a,k) = \frac{1+p_1(a)k^2}{p_2(a)+p_3(a)k^2} \tag{190}$$

Though it seems that the k-dependent terms are generally hard to detect.

- Simpson plots. mu gamma flow chart.
- Pro: model-independence, general constraints. Con: hard to pick a good ansatz, lost direct connection to action. Linear PT only.

4.2 CMB

- The CMB is one of the most powerfully constraining data sets for cosmological parameters. This is because it has lots of features (peak heights, widths, positions) that can be calculated to a very large extent using linear theory and 'known'/relatively simple physics, e.g. Thomson scattering. Compare this to probes of large-scale structure, which require nonlinear modelling and lots of prescriptions to fill in things we aren't 100% sure how to describe, e.g. SN feedback, magnetic fields, etc.
- However, most of the features above multipole $\ell=100$ were laid down at early times in the universe, long before most MG/DE models have effects. In fact, only the low-ℓ CMB tail is sensitive to the late-time universe. This part is called the Integrated Sachs-Wolfe plateau, or ISW plateau.
- The height and shape of the ISW plateau is set by the integrate time variations of the lensing potential, i.e. $\Phi_{\rm lens}=(\Phi+\Psi)/2$. So in the model-independent framework above, it is controlled by (variations of) the Σ function.

- On it's own, the ISW effect only rules out extreme models with rapid growth of structure, because on those large scales cosmic variance limits the minimum error bar of CMB measurements. However, it did prove to be extremely useful for tests of the cubic Galileon, with $c_2 = -1$ and c_3 a parameter to be constrained.

$$C_\ell^{Tg} = 4\pi \int \frac{dk}{k} \Delta_\ell^{ISW} \, \Delta_\ell^g \mathcal{P}_\mathcal{R}(k) \tag{191}$$

$$\Delta_\ell^{ISW} = \int_{\tau_*}^{\tau_0} d\tau (\Phi' + \Psi') \, j_\ell(k[\tau_0 - \tau]) \tag{192}$$

where P is the power spectrum of the primordial curvature perturbation. $\rightarrow$ Renk plot. note that for the quartic and quintic models things are acceptable, but for the cubic Galileon the ISW contribution is large and has completely the wrong sign. This makes it incompatible with the data points (but a positive sign would not have), which come for cross-correlating with the WIS galaxy survey.

The best-fit cubic galileon model is in a 7.8 sigma tension with this data.

- CMB lensing also contributes secondary anisotropies. Since MG can strongly affect the LSS CMB photons propagate through, this (low-ℓ) affect is also a probe, see plot.

4.3 Matter Power Spectrum, RSDs & Growth

- Consider the **autocorrelation** function between density fluctuations at two different locations, for $\mathbf{r} = \mathbf{x} - \mathbf{x}'$:

$$\xi(r) = \langle \delta(\mathbf{x}) \delta(\mathbf{x}') \rangle = \frac{1}{V} \int d^3\mathbf{x} \, \delta(\mathbf{x}) \delta(\mathbf{x} - \mathbf{r}) \tag{193}$$

$\rightarrow$ measure of structure separated by r averaged over survey volume. The power spectrum is effectively the Fourier transform of the autocorrelation function:

$$\left\langle \tilde{\delta}(\mathbf{k}) \tilde{\delta}(\mathbf{k}') \right\rangle = (2\pi)^3 P(k) \delta^{(3)}(\mathbf{k} - \mathbf{k}') \tag{194}$$

$$\xi(r) = \int \frac{d^3k}{(2\pi)^3} P(k) e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} \qquad P(k) = \int d^3r \, \xi(r) e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} \tag{195}$$

The Poisson equation gives us (dropping tildes again):

$$\delta_m(k,a) = -\frac{2k^2 a}{3\Omega_{M0} H_0^2} \Phi(k,a) \tag{196}$$

$$\Rightarrow \langle \delta(\mathbf{k})^2 \rangle \propto k^4 \langle \Phi^2(k,a) \rangle \propto k^4 D(a)^2 T(k)^2 \langle \Phi_0^2(k) \rangle \tag{197}$$

$$\propto \left(\frac{k}{H_0} \right)^{n_s} D(a)^2 T(k)^2 \qquad \text{where} \qquad P_\Phi^0(k) \propto \frac{k^{n_s - 1}}{k^3} \tag{198}$$

The power spectrum has dimensions of length^3. At low k the k^{n_s} causes the power spectrum to grow, but at large k the transfer functions damp it out. The peak is at around $k \sim 2 \times 10^{-2}$, and is related to the time of matter-radiation equality.

- However, we don't observe the matter power spectrum directly, for two reasons. Firstly, we don't see dark matter; instead we see fluctuations in *galaxy* density. This is related to DM density fluctuations by *bias*. On large, linear scales, the bias is approximate as redshift-dependent only, $\delta_m = b(z)\delta_g$. On smaller scales this assumption breaks down. (Issue in MG – how does b change in other gravitites?)

- The second reason is because we observe galaxies in redshift space. We don't measure distances directly, but instead we obtain a redshift measurement and convert it to a distance assuming a cosmology. This would work fine if everything was comoving perfectly, but in reality galaxies have motions that cause us to mis-estimate their distances. These **redshift space distortions** (RSD) turn out to be an excellent probe of gravity (though extracting the measurements requires a fair amount of GR assumptions). $\rightarrow$ RSD and FoG diagrams. Two points in redshift and real space are related by

$$\mathbf{s}(\mathbf{r}) = \mathbf{r} + v_r(\mathbf{r})\hat{\mathbf{r}} \tag{199}$$

where $v_r(r)$ is the PV projected in the radial direction (and recall $c = 1$ here).

- Kaiser 1987 (see also Hamilton 1998) showed that, where μ is the cosine between the vector $\mathbf{k}$ and the line of sight, i.e. $\mu = \mathbf{k}\cdot\hat{\mathbf{r}}/k$:

$$\delta_g^s(k) = \left(1+\beta(z)\mu^2\right)\delta_g^r(k) \qquad \beta(z) = \frac{f(z)}{b(z)} \tag{200}$$

$$\Rightarrow P_g^s(k,\mu,z) = b(z)^2\left(1+\beta\mu^2\right)^2 P_m^r(k,z) \tag{201}$$

$$P_g^s(k,\mu,z) = \left(b(z)+f(z)\mu^2\right)^2 P_m^r(k,z) \tag{202}$$

The small scale FoG effect is usually modelled as a multiplicative factor that brings in another free parameter, the velocity dispersion σ_v:

$$F_L(k,\mu^2) = \left[1+(k\mu\sigma_v)^2\right]^{-1} \qquad F_G(k,\mu^2) = \exp\left[-(k\mu\sigma_v)^2\right] \tag{203}$$

Note that these are just models – they are not fundamentally derived expressions! The redshift space power spectrum is usually decomposed into even multipoles. The monopole and quadrupole are commonly used, the hexadecapole is sensitive to nonlinear scales and requires more in-depth modelling. → Show RSD figs, fsig8 plots. Note that f is degenerate with the overall amplitude of the power spectrum σ_8, hence we measure combination best.

- Chosen not to discuss BAO in detail here as usually consider geometric probes, though ofc useful for background EoS.

4.4 Codes for Calculating Linear Perturbations in MG

- Einstein-Boltzmann solvers evolve the system of background and linearised Einstein field equations and conservation equations for matter and radiation of produce accurate outputs of the simplified cases we have been discussed. They can handle advanced extensions like curvature, massive neutrinos, different conditions for inflation that we couldn't treat analytically. Their results are valid on large horizon scales, but can't be trusted fully on nonlinear scales ($k \gtrsim 0.1$ h/Mpc).

- Usual outputs are CMB power spectra, including TT, TE, EB, etc.; matter power spectra; CMB lensing potential, etc. They are fast enough (usually) to be run tens of thousands of times, allowing MCMC analysis to constrain cosmological parameters.

- There are two dominant packages on the market for LCDM: CAMB (Fortran, synchronous gauge) and CLASS (C). Modified code versions incorporating MG effects now exist, some for specific MG models, and some for general parameterised frameworks or several models. Based on CAMB: MGCAMB (mu-sigma-gamma), ISiTGR, and EFTCAMB based on CAMB. hi_class based on CLASS, and a few other independent codes such as DASh, COOP. I'm aware of specific codes for f(R) gravity, Galileons, and Einstein-Aether at the least.

- Bellini comparison plots.

- EFTCAMB and hiclass are two of the most popular and well-tested; they employ different action-level parameterisations. I use hiclass a lot for Horndeski analysis; EFTCAMB employs a slightly more general framework that uses different notation. On the other hand, if you're doing a lensing analysis and really just want to constrain Σ, something simpler like MGCAMB or ISiTGR may be your best bet.

4.5 Further GW Bounds

- GW propagation equation on FRW:

$$h_A'' + (2+\nu)\mathcal{H}h_A' + \left(c_T^2k^2 + a^2m^2\right)h_A = \Gamma_A \tag{204}$$

In Horndeski, $\nu = \alpha_M$ and the other parameters are zero. The modified solution is related to the GR one by (Nishizawa 2017):

$$h_{MG} = e^{-\mathcal{D}}e^{-ik\Delta T}h_{GR} \tag{205}$$

$$\mathcal{D} = \frac{1}{2}\int_0 dx\,\nu(x) \qquad \Delta T = \int_0^{z_*} dz\,\frac{1}{\mathcal{H}}\left[\frac{\nu}{1+z} - \frac{m^2}{2k^2(1+z)^3}\right] \tag{206}$$

- Focussing on just ν for now, this term affects the effective luminosity distance of GW source:

$$h \propto \frac{1}{d_L} \qquad \Rightarrow \qquad h \propto \frac{e^{-\mathcal{D}}}{d_L} \equiv \frac{1}{d_{GW}} \tag{207}$$

$$d_L = c(1+z)\int_0^{z_*} \frac{1}{H} dz \tag{208}$$

If we can get both a redshift and good GW measurement from a standard siren, we can potentially bound ν this way. The measurement would work best with a high redshift source, e.g. a MBHB from LISA. → plots from Belcagem et al., who define:

$$\frac{d_{GW}(a)}{d_L(a)} = \Xi_0 + a^n(1-\Xi_0) \tag{209}$$

- This measurement may not work in Horndeski (Dalang & Lombriser 2019).

4.6 Voids

- Voids are a particularly interesting environment in which to test gravity. Their low densities mean that screening mechanisms are not expected to activate. Voids are only recently being developed as an observational probe, because by their very nature they are hard to detect; high spectroscopic density is needed to accurately build up a picture of their true shape. Most often void signals are boosted by stacking.

- MG can affect voids in three ways:

  1. It can affect the lensing potential of the void by causing $\Phi \neq \Psi$ as above in cosmology;
  2. The density profile of the void can be affected if the scalar field (say) contributes a non-negligible energy density.
  3. Changes to the background expansion rate (and fifth forces) may affect the rate at which large voids expand and cannabilise small voids, i.e. affecting population statistics.

- A commonly-used DM density profile in LCDM is:

$$\delta(r) = \delta_c \left[\frac{1-\left(\frac{r}{r_S}\right)^\alpha}{1+\left(\frac{r}{r_V}\right)^\beta}\right] \tag{210}$$

where δ_c is the central density, r_s is a scale radius that controls where the void crosses zero (roughly), and α and β control the steepness of the void walls and its compensation ridge. (In fact two of the parameters are generally fixable in terms of the other two.)→ plots of void profiles.

- In reality voids are found in biased tracers. However, eq.(210) is sometimes called the 'universal' void profile, as it seems to do a good job of modelling the DM voids found in most tracers. Most voids are also shown to be 'self-similar', meaning this profile shape seems to apply to a broad range of void sizes (though larger voids are of course generally deeper).

- Typical depths range down to $\delta_v \sim -0.8$, and radii $r_V \sim 10-70$ Mpc.

- Real voids are of course not circular, but they become so when stacked. As a general rule, real non-circular voids and void populations require simulations to study. A number of void-finding techniques exist.

- Without simulations, we cannot study ii) or iii) easily. However, we can posit that DE density have a small (negligible) shape on the void profile, and calculate the scalar profile, fifth force and lensing over that void. A well-studied example is cubic Galileon voids, however there seems to be an unphysicality here – perhaps caused by a breakdown of the QSA? → plots.

- Another interesting related phenomenon is troughs – cylindrical underdensities along the line of sight. Arguably not physical, but look for asymmetry in lensing signal (nothing statistically significant found yet) → Gruen plots, DES.

5 Final Comments

- The past ten years have seen rapid progress in creating, testing, and ruling out (or not) models of gravity. The basic motivators haven't changed (acceleration, agnosticism and anomalies), but we've learnt a lot about the types of physics that do and don't work, and even explored new territory (screening, parameterisations, bigravity).
- Surveys are ongoing and still to come, with tests of DE/MG as one of their key science drivers (DES, DESI, LSST). We're going to need to get more advanced in our modelling of the nonlinear regime, both in LCDM and MG, and more simulations are needed (though these are not a panacea).
- GWs have undoubtedly been the poster child of the past 3 years or so, and will continue to yield results as we climb from first detections to the observatory era.
- Things largely omitted from these notes: many individual gravity theories, unified dark sector theories, lensing, BAO, simulations, nonlinear prescriptions, EFTofDE details, bias details, Bayesian analysis...